The Prolate Spheroidal Transform for Gated SPECT

B. Feng, *Student Member, IEEE*, A. Sitek, *Member, IEEE*, and G. T. Gullberg, *Senior Member, IEEE*

***Abstract*—In this paper, we propose a method to calculate the volume and surface area of the left ventricle (LV) in the prolate spheroidal coordinate system. The prolate spheroidal coordinate system is well suited for use with the LV due to the approximately ellipsoidal shape of the LV. The prolate spheroidal transform was applied to short axis gated SPECT data, and the volumes of the LV were calculated from the mid-myocardium surface. Also, we calculated the areas of the mid-myocardial surface. The information obtained may be used to estimate the LV wall thickening during a cardiac cycle. This is possible due to the fact that the myocardium is almost incompressible. The method was verified by performing tests using the MCAT phantom. The calculations of the LV volumes and surface areas of the MCAT phantoms agreed with the true values. The prolate spheroidal transform was also applied to patient gated SPECT data. Strong correlations were evident between the calculated volume and surface area values in MCAT simulations and in the patient study. These results imply that the calculation of the surface area, which is more convenient for hearts with defects, may be an alternative way to estimate the volumes of the myocardium.**

***Index Terms*—Gated cardiac SPECT, LV wall thickness, myocardial volume, prolate spheroidal transform.**

I. INTRODUCTION

PRIOR knowledge of the geometry of the left ventricle (LV) is important for developing mathematical models to investigate the global function [1], [2] and for segmenting the LV [3]–[5]. Initially, simple geometries, such as spherical, cylindrical, and ellipsoidal, were assumed for the LV. In recently developed models [6], [7], the geometry of the LV was assumed to be a truncated thick-walled ellipsoid with confocal or nonconfocal surfaces (see Fig. 1). Under such an assumption about the shape of the LV, the use of the ellipsoid coordinate system is convenient. The prolate spheroidal transform is defined such that it transforms the spatial activity distribution of the LV from the Cartesian coordinates (x, y, z) to the ellipsoid coordinates (ξ, θ, ϕ) according to the following:

$$x = C\sinh(\xi)\sin(\theta)\cos(\phi) \quad (1)$$

$$y = C\sinh(\xi)\sin(\theta)\sin(\phi) \quad (2)$$

$$z = C\cosh(\xi)\cos(\theta) \quad (3)$$

where C is the focal length for the prolate spheroidal system.

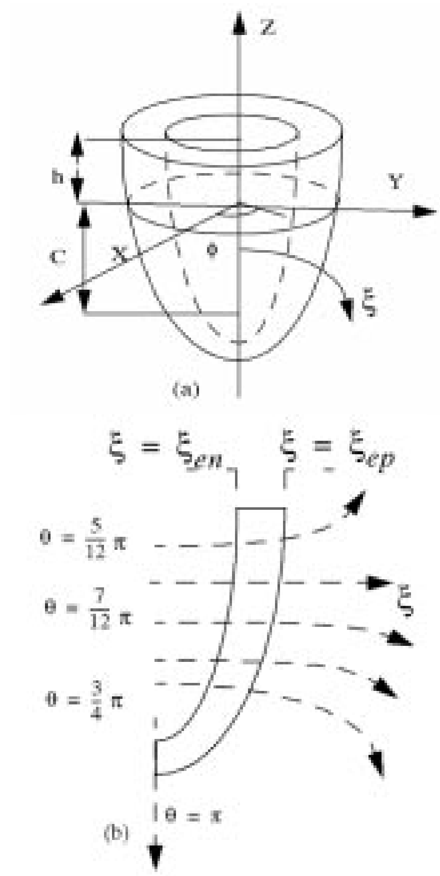

Fig. 1. (a) Geometry of the ellipsoidal model of the LV showing the focal length C and the truncation height h above the equator. (b) Cross-section through the LV showing coordinates ξ and θ.

For the MCAT phantom with perfect confocal ellipsoid shapes, if transformed with the correct focal length C, the endocardial and epicardial surfaces of the LV become straight planes [see Fig. 2(a)] in the prolate spheroidal coordinate system. Even when using real patient data that are transformed with an approximate C, the transformed images of the LV resulted in a simple geometry with approximated plane surfaces. This suggests that there is the possibility of segmenting the left ventricle in the ellipsoid coordinate system since the surface detection may be more easily implemented than in the Cartesian coordinate system. In this paper, we focus on the idea of calculating the volume and surface area of the LV in the ellipsoid coordinate system, thereby avoiding the complexity involved in edge detection. We found the maximum activity layer in the prolate system to be the mid-myocardium. Therefore, we calculated the surface area of the mid-myocardium and the volume it enclosed. During the prolate spheroidal transform of the data, a trilinear interpolation was used.

Manuscript received June 8, 1999; revised December 28, 2000. This work was supported in part by the National Institutes of Health under Grant RO1 HL39792 and in part by Marconi Medical Systems, Inc.

B. Feng and G. T. Gullberg are with the Medical Imaging Research Laboratory, University of Utah, Salt Lake City, UT 84108 USA (e-mail: bfeng@doug.med.utah.edu; ggullbe@hsc.utah.edu).

A. Sitek was with the Medical Imaging Research Laboratory, University of Utah, Salt Lake City, UT 84108 USA. He is now with E.O. Lawrence Berkeley National Laboratory, Berkeley, CA 94720 USA (e-mail: Asitek@lbl.gov).

Publisher Item Identifier S 0018-9499(01)05270-4.

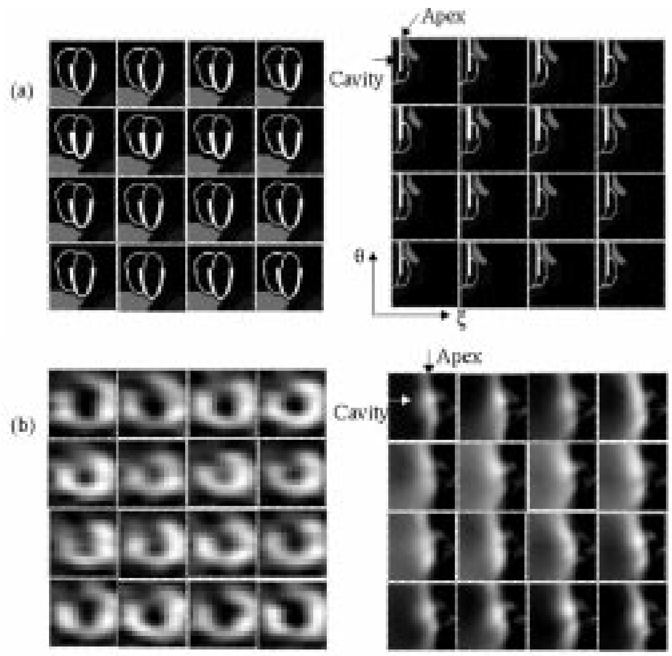

Fig. 2. (a) For a 16-frame MCAT phantom that simulates a whole cardiac cycle, the original images (left) and transformed images in the ellipsoid system (right). (b) For a set of gated SPECT data that consists of 16 frames, the original images (left) and transformed images (right).

Faber *et al.* [2], [3] used a mixture of the cylindrical and spherical coordinate system to detect the surface of the LV and calculate the ejection fraction (EF). While the cylindrical coordinate system is used to describe the base and midventricular section of the LV, the spherical coordinate system is used for the apex region. However, the switch from one system to another seems arbitrary. Declerck *et al.* [8] also used an unconventional coordinate system to register and align SPECT images. Compared with the approaches by others, the greatest advantage of using the ellipsoid coordinate system in our volume and area calculations is its simplicity in implementation.

II. Methods

To calculate the LV volume and surface area in the ellipsoid coordinate system, the Jacobian associated with the coordinate transformation [(1)] must be calculated.

The volume of the LV is calculated using

$$V = \iiint_V j \, d\xi \, d\theta \, d\phi \tag{4}$$

with J being the determinant of the Jacobian matrix defined as

$$J = \det \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{bmatrix}. \tag{5}$$

So, using (1), we have

$$J = C^3 (\sinh \xi)^3 \sin \theta (\cos \theta)^2 + C^3 (\cosh \xi)^2 \sinh \xi (\sin \theta)^3. \tag{6}$$

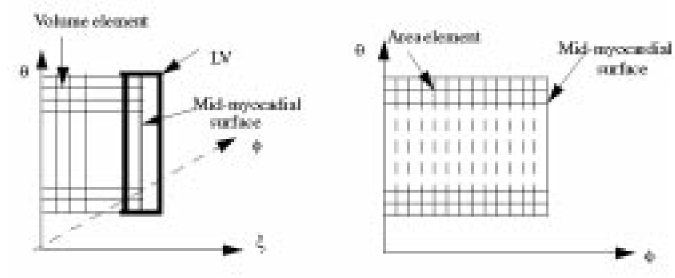

Fig. 3. The symbolic drawing of the ranges of integration and elements used in the volume and area calculations.

Similarly, for the calculation of the surface area of the LV

$$S = \iint_S \sqrt{x^2 + y^2} \, d\phi \, dz = \iint_S \sqrt{x^2 + y^2} J_{2D} \, d\theta \, d\phi. \tag{7}$$

We calculated the corresponding Jacobian

$$J_{2D} = \det \begin{bmatrix} \frac{\partial \phi}{\partial \theta} & \frac{\partial \phi}{\partial \phi} \\ \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{bmatrix} = C \cosh \xi \sin \theta \tag{8}$$

and arrived at

$$S = \iint_S C^2 (\sinh \xi)(\cosh \xi)(\sin \theta)^2 \, d\theta \, d\phi. \tag{9}$$

To calculate the volumes, the range of integration for both the volume and area calculations needed to be specified. In this paper, the mid-myocardium was the surface used for the surface and volume calculations. Due to the finite resolution of gated cardiac SPECT, the maximum activity occurs near the mid-myocardium. The location of the mid-myocardium can be found easily by locating the maximum activity on the ellipsoid coordinates. We calculated the surface area of the mid-myocardium and the volume enclosed by it. The integrations [(2) and (5)] were made using a finite sum method. Fig.3 shows the integration elements and ranges used in these calculations.

III. Results

We used an MCAT phantom to test our method. We performed the calculations for the MCAT phantom studies under various conditions: with and without noise (Fig. 4) and changing the focal length and translating the left ventricle in the long axis (z) direction. In each case, we obtained encouraging results. We also found strong correlations between the calculated values of the volume and the area (Fig. 5), which suggests that there is a way to calculate the volume of the LV with large defects indirectly by calculating the area, since the area calculation is less affected by defects.

We then applied this method to gated SPECT data, calculating the mid-myocardial volume and area (Figs. 7 and 8). We found the calculated values correlated with the reasonable time-varying shape during a cardiac cycle, which indicated that the LV contracted faster than it expanded. A strong correlation was also found between the volume and area calculations (Fig. 6).

The average wall thickness of the LV can be defined as the volume of the myocardium divided by the mid-ventricular area.

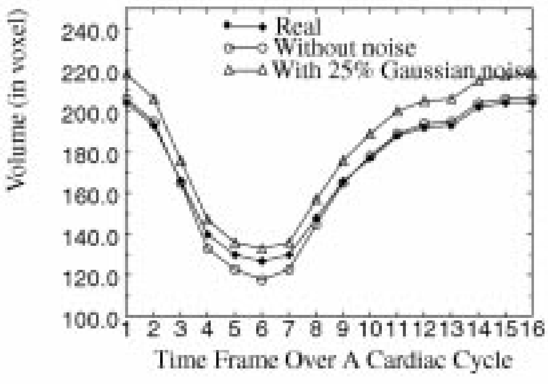

Fig. 4. Calculated mid-myocardial volumes for the MCAT phantom with and without adding noise, compared with real values.

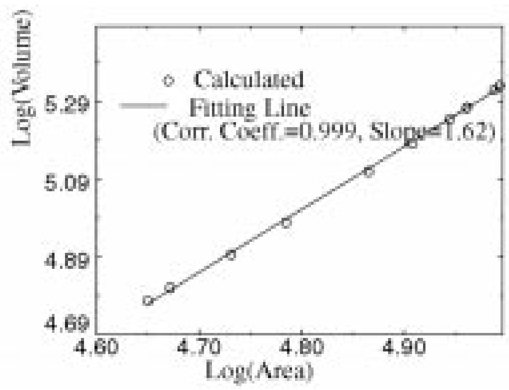

Fig. 5. The log–log plot of the calculated mid-myocardial volumes versus areas for the MCAT phantom over a cardiac cycle.

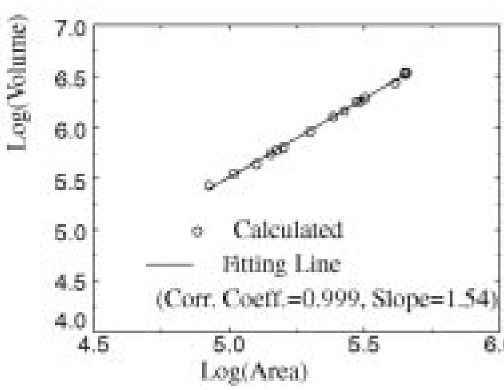

Fig. 6. The log–log plot of the calculated mid-myocardial volumes versus areas for the gated SPECT data over a cardiac cycle.

Due to the incompressibility of the myocardium, the average wall thickness of the LV is inversely proportional to the midventricular area. The average wall thickness of the LV of the MCAT phantom is compared to its actual value in Table I. In the calculation of the average wall thickness of the LV, the volume of the myocardium is known from the log file of the MCAT phantom. The only factor that affects the calculation of the average wall thickness is the calculation of the midventricular area.

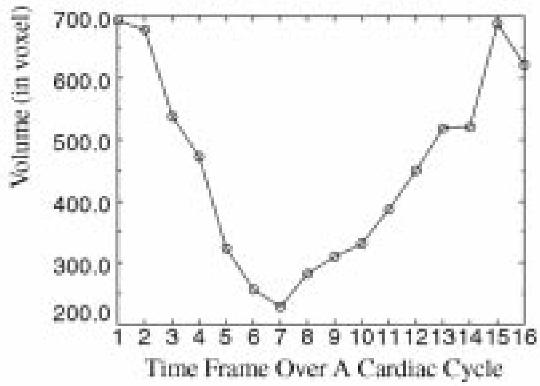

Fig. 7. The calculated mid-myocardial volumes over 16 time frames in a cardiac cycle for the gated SPECT data.

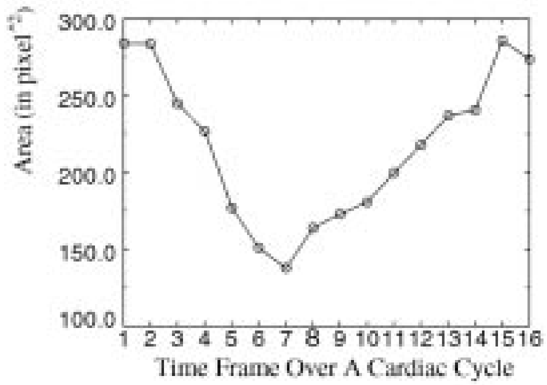

Fig. 8. The calculated mid-myocardial areas over 16 time frames in a cardiac cycle for the gated SPECT data.

TABLE I
THE LV VOLUME, SURFACE AREA, AND WALL THICKNESS OF THE MCAT PHANTOM

Volume, area and wall thickness	End-Diastole	End-Systole
Myocardium volume V	$144ml$	$145ml$
Middle layer area A (real value)	$151cm^2$	$105cm^2$
Calculated middle layer area A	$148cm^2$	$107cm^2$
Average wall thickness calculated by V/A (for real V and calculated A)	$1.02cm$	$1.36cm$
Average wall thickness calculated by V/A (for real V and A)	$0.95cm$	$1.38cm$

IV. DISCUSSION

A way to calculate the left ventricle volume and area and track the LV movement was described. Since the ξ axis is approximately perpendicular to the LV surfaces, it is advantageous to use the ellipsoid coordinate system because small elements used in the integrations fit the geometry well, even in the apex region, where the cross-section size of the LV changes rapidly between adjacent layers. Though the method is model-based in nature, the calculations are insensitive to the chosen value of the focal length and to the alignment of the images of the left ventricle in the Cartesian coordinate system. While the focal length used in the transform was changed over a large range (from 17 to 37 pixels, with the exact value being 27 pixels), and the short axis image was translated along the axial direction over a range of

20 pixels, the calculated volumes for the MCAT phantom were correct to within 5%. Mathematically, the calculations of the volume and area can be carried out in any coordinate system if the associated Jacobian matrix and the range of integration are correctly determined. We prefer using the ellipsoid coordinate system instead of the Cartesian coordinate system because the ellipsoid coordinate system is better for determining the integrating region in the volume and area calculations. Since the geometry of the LV can be approximated by a truncated, confocal, ellipsoidal model, by choosing the optimal parameters [9] (the focal length, the alignment of the LV in the ellipsoid coordinate system), the geometry of the LV can be described approximately by ellipsoid coordinates (ξ, θ, ϕ) bounded by: $0 \leq \phi < 2\pi, \theta_{\min} \leq \theta \leq \pi$, and $\xi_{\min} \leq \xi \leq \xi_{\max}$ (see Fig. 1). Here $\xi_{\min}$ and $\xi_{\max}$ occurred at the endocardial and epicardial surfaces, respectively. Though therange of integration is dependent on θ and ϕ for gated cardiac SPECT, we assume they change slowly enough over space to enable us to develop future algorithms that can effectively detect and interpolate the LV surfaces.

The strong correlation between the LV volume and area values in a cardiac cycle (see Figs. 5 and 6) suggests that there is a way to calculate the volume indirectly. This can be done by calculating the area when large defects exist in some frames of images since the area calculation is less sensitive to the image defects. Choosing a sphere as an example, if the image of the sphere was truncated by one half of its size, the direct volume calculation will give a 50% error. Whereas the error of calculating the surface area is 25%. The log-log plots in Figs. 5 and 6 show that the data fits well to lines with different slopes, which implies that the hearts were beating in a pattern to keep their "shape" unchanged in a topological sense. Considering an object with a cubic or spherical shape, when it expands and contracts, and while keeping its shape unchanged, its volume and area will fall on the line with the slope of 3/2 in the log–log plot.

Since the myocardium can be regarded as almost incompressible, the midventricular area calculation can be used to estimate the average thickness of the LV wall for each frame of gated SPECT data, which in turn may indicate how well the heart is performing. If combined with some image registration algorithm, the area calculation can be applied locally to calculate the local wall thickening of the LV.

Having established an estimation of the wall thickness, the detection of the midventricular surface can be used to generate a fine mesh for the finite-element analysis of the LV.

ACKNOWLEDGMENT

The authors would like to thank P. Christian for his help in providing the gated SPECT data and S. Webb for proofreading the manuscript.

REFERENCES

[1] M. G. J. Arts, "A mathematical model of the dynamics of the left ventricle and the coronary circulation," Ph.D. defense, 1978.

[2] T. L. Faber, C. D. Cooke, R. D. Folks, J. P. Vansant, K. J. Nichols, E. G. Depuey, R. I. Pettigrew, and E. V. Garcia, "Left ventriclular function and perfusion from gated SPECT perfusion images: An integrated method," *J. Nucl. Med.*, vol. 40, no. 4, pp. 650–659, Apr. 1999.

[3] T. L. Faber, E. M. Stokely, R. M. Peshock, and J. R. Corbett, "A model-based four-dimensional left ventricular surface detector," *IEEE Trans. Med. Imag.*, vol. 10, pp. 321–329, Sept. 1991.

[4] G. Germano, H. Kiat, P. B. Kavanagh, M. Moriel, M. Mazzanti, H. Su, K. F. Van Train, and D. S. Berman, "Automatic quantification of ejection fraction from gated myocardial perfusion SPECT," *J. Nucl. Med.*, vol. 36, no. 11, pp. 2138–2147, Nov. 1995.

[5] E. G. DePuey, K. Nichols, and C. Dobrinsky, "Left ventricular ejection fraction assessed from gated Technetium-99m-Sestamibi SPECT," *J. Nucl. Med.*, vol. 34, no. 11, pp. 1871–1876, Nov. 1993.

[6] P. H. M. Bovendeerd, T. Arts, J. M. Huyghe, D. H. Van Campen, and R. S. Renekman, "Dependence of local left ventricular all mechanics on myocardial fiber orientation: A model study," *Biomechanics*, vol. 25, no. 10, pp. 1129–1140, Mar. 1992.

[7] D. D. Streeter Jr. and W. T. Hanna, "Engineering mechanics for successive states in canine left ventricular myocardium," *Circ. Res.*, vol. 33, pp. 639–655, Dec. 1973.

[8] J. Declerck, J. Feldmar, M. L. Goris, and F. Betting, "Automatic registration and alignment on a template of cardiac stress & rest SPECT images,", INRIA Res. Rep., Jan. 1996.

[9] B. Feng, A. Sitek, E. V. R. Di Bella, and G. T. Gullberg, "A 3D prolate spheoidal transformation for tracking and analyzing of the LV motion," in *JNM Proc. 46th Annu. Meeting Nuclear Medicine*, 1999, p. 184.